

Quiz 23

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. Today, we will walk through conditional expectation using ideas from dilution and mixing in lecture.

1 Dilution

It is now year 3000 where watermelon is a sacred fruit. Everyone receives N watermelons at birth. However, citizens of this future must participate in the watermelon ceremonies, annually. At this ritual, citizens can choose to pick 1 melon at random, to replace with cantaloupes.

1. Given that a citizen has m watermelons at the n th year, what are all the possible number of watermelons that this citizen can have in the $n + 1$ th year, and what is the probability that each of these situations occur?
2. Let us suppose (for this part only) that a citizen now picks **two** watermelons at random, at this ritual. Given that a citizen has m watermelons at the n th year, what are all the possible number of watermelons that this citizen can have in the $n + 1$ th year, and what is the probability that each of these situations occur?
3. Again, let us consider the original scenario, where each citizen picks only 1 melon at random at the ritual. Given that a citizen has m watermelons at the n th year, how many watermelons will a citizen then have in year $n + 1$, on average?
4. After n years, compute the average number of watermelons a particular citizen will have left.

5. If all citizens begin with 100 watermelons, after how many years will citizens in this society end up with 99 cantaloupes?