Quiz 16 Solutions

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. "Intuition Practice" might be tricky; watch out for subtleties. "Proofs" will be challenging to start; develop an arsenal of *approaches* to starting a problem.

1 Intuition Practice

For each of the following, circle "Always True" or "False". Then if false, construct two events X and Y that disprove the condition, given a uniform distribution across $\Omega = \{1, 2, 3, 4, 5, 6\}$.

1. (Always True or False) If $P[X] > P[Y]$, then $P[X|Z] > P[Y|Z]$.

   Solution: False.

   $X = \{1, 2, 3\}$, $Y = \{3, 4\}$, $Z = \{3, 4\}$.

   $P[X] = \frac{1}{2}, P[Y] = \frac{1}{3}$, so $P[X] > P[Y]$.

   $P[Z] = \frac{1}{3}, P[X, Z] = \frac{1}{3}, P[Y, Z] = \frac{1}{3}$, so $P[X|Z] = \frac{1/3}{1/3} = 1, P[Y|Z] = \frac{1/3}{1/3} = 1$ and $P[X|Z] = P[Y|Z]$.

2. (Always True or False) If X is independent of Y, $P[X] = \sum_z P[Z, X|Y]$.

   Solution: True. We sum out over z to get $\sum_z P[Z, X|Y] = P[X|Y]$. Since X is independent of Y, we know $P[X|Y] = P[X]$.

3. (Always True or False) $P[X] > P[Y]$, then $P[XZ] > P[YZ]$.

   Solution: False.

   We proceed with the same example from the first part.

   $P[Z] = \frac{1}{3}, P[X, Z] = \frac{1}{3}, P[Y, Z] = \frac{1}{3}$, so $P[XZ] = P[Y|Z]$.