Quiz 17 : Linear Regression Solutions

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam.

1 Concepts

1. (**True** or **False**) Given some linear estimate of $L[Y|X] = aX + b$, we have that $E[(Y - aX - b)X] = 0$. Provide justification.

   Solution: True. This is the projection property.

   We know that the error $E[Y - E[Y]] = 0$. Likewise, for $E[Y - L[Y|X]] = 0$, so $E[(Y - L[Y|X])X] = 0$.

2. (**True** or **False**) For any random variables X, Y, Z, where X, Z are independent and $X = \alpha Y + Z$, the signal-to-noise ratio (SNR) is $\frac{\alpha^2 E[Y^2]}{\sigma^2}$, where $\text{var}(Z) = \sigma^2$. (αY is our signal and Z is our noise.)

   Solution: False. Y and Z must be zero-mean. Signal-to-noise ratio is formally defined to be $\frac{\sigma^2_{signal}}{\sigma^2_{noise}}$. Since σ^2 is variance, we have that

   $$SNR = \frac{\text{var}(\alpha Y)}{\text{var}(Z)}$$

   Since both Y and Z are zero-mean, we have the following.

   $$SNR = \frac{E[(\alpha Y)^2] - (\alpha E[Y])^2}{\sigma^2} = \frac{\alpha^2 E[Y^2]}{\sigma^2}$$

2 Quantities

For some large n, take n points along the unit square centered at the origin. (The corners of the unit square are $(1, 1), (1, -1), (-1, -1), (-1, 1)$). Compute $L[Y|X]$.

Solution: Note that $\text{cov}(X, Y) = 0$, so $L[Y|X] = E[Y]$, where $E[Y] = 0$. Thus, $L[Y|X] = 0$.