

# Quiz 4

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**This quiz does not count towards your grade.** It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam.

## 1 Graph Theory

For each of the following, consider a graphs without self-loops or multi-edges. For each True or False question, provide a brief justification.

1. (True or False) The sum of all degrees in an undirected *tree* with  $n$  vertices is  $2(n - 1)$ .
2. (Short Answer) Take a binary tree with  $n$  vertices, where for some  $k \in \mathbb{Z}$ ,  $n = \sum_i^k 2^i$ . (i.e., enough vertices for a complete tree) Find the longest path, and remove all of its edges. The resulting graph has *at most* how many degrees?
3. Given a tree with undirected edges, prove that each vertex  $u$  has exactly one path to any other vertex  $v$ . You may use any of the other definitions of a tree.