Quiz 3 Solutions

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam.

1 Stable Marriage Algorithm

For each of the following, prove or disprove the statement. Let TMA be the traditional stable marriage algorithm. Let each instance involve n men and n women.

1. Consider an instance, where preference lists may indicate "equality" between two choices. TMA will still produce stable pairings. (When W is presented with two men where $M_1 = M_2$, W will pick arbitrarily if necessary.)

   Solution: True. Arbitrarily assign an ordering for any $M_1 = M_2$, so that $M_1 > M_2$ or $M_2 > M_1$, and run TMA.

2. If $n > 2$, $\forall k < n$, it is possible construct an instance that takes TMA exactly k days to terminate.

   Solution: True. Construct a solution that takes n days. One example would be to setup all n women with identical preferences. Let them all be ordered $M_1, M_2 ... M_n$. Also setup all n men with identical preferences. Let them all be ordered $W_1, W_2 ... W_n$. We have that this solution takes n days to terminate. To get $n-1$ days, change one man's preferences - say M_2's - so that his top choice is W_2 instead of M_2. We can continue this do this until termination in 1 day.

3. Given TMA takes $k \leq n$ days, there can be no more than $\frac{(n-1)n}{2} - \frac{(n-k)(n-k-1)}{2}$ rejections.

   Solution: True. Note that $\frac{(n-1)n}{2} - \frac{(n-k)(n-k-1)}{2} = \sum_{i=1}^{n} - \sum_{i=1}^{n-k-1} = \sum_{i=n-k}^{n-1} i = (n-1) + (n-2) + \cdots + (n-k)$. Note that the most rejections that can occur on day 1 is $n-1$, where all men propose to one woman. For the same reason, the maximum on the next day is $n-2$, then $n-3$ and so on and so forth, until $n-k$.