# Crib 13 : Bayes' Rule, Independence

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

## 1 Bayes' Rule

- Bayes' Rule follows from both the definition of conditional probability and the chain rule.

$$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B)}$$

- Keep in mind that Bayes' Rule is not always the easiest to reason about. We can alternative reason about the definition of conditional probability:

$$\Pr(A|B) = \frac{\Pr(A, B)}{\Pr(B)}$$

  If $\Pr(B)$ is complex, we can also consider expanding it using the law of total probability.

$$\Pr(A|B) = \frac{\Pr(B|A)\Pr(A)}{\Pr(B|A)\Pr(A) + \Pr(B|\bar{A})\Pr(\bar{A})}$$

## 2 Independence

Do not forget that the equations specified below are true **if and only if** the events are pairwise independent or mutually independent, respectively.

- Given two events $A$ and $B$, pairwise independence states that the two following statements are two, where the second follows from the first. (Apply Bayes')

$$\Pr(A, B) = \Pr(A) \Pr(B)$$
$$\Pr(A|B) = \Pr(A)$$

- Given three events $A$, $B$, and $C$, mutual independence states the three are events **1.** are pairwise independent and **2.** satisfy the following property.

$$\Pr(A, B, C) = \Pr(A) \Pr(B) \Pr(C)$$

- Two events are **disjoint** if $A \cap B = \emptyset$. Independence does *not* imply the events are disjoint, and disjoint events are not necessarily independent. For example, say $A$ occurs only if $B$ does not, and $A$ occurs with non-zero probability. Then $\Pr(A|B) = 0 \neq \Pr(A)$, and two disjoint events $A$ and $B$ are not independent.