

# Crib 9

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

## 1 Countability

- A function is **injective**, if every  $x$  maps to a unique  $y$ . More formally,  $\forall x, y, (f(x) = f(y)) \implies x = y$ .
- A function is **surjective**, if every  $y$  has an  $x$  mapping to it. Let  $A$  be the domain and  $B$  be the co-domain.  $\forall b \in B, \exists a \in A, f(a) = b$ .
- A **bijection** is both injective and surjective.
- A set is **countable** if there exists a bijection between it and the naturals  $\mathbb{N}$ .
- Countable is synonymous with denumerable and enumerable; it effectively means there exists some ordering of all elements in the set.
- The set of all integers ( $\mathbb{Z}$ ), naturals ( $\mathbb{N}$ ), and rationals ( $\mathbb{R}$ ) are countable.
- The set of all irrationals ( $\mathcal{Q}$ ) and reals ( $\mathbb{R}$ ) uncountable.
- The power set of a set  $S$  is the set of all possible subsets, without repeating elements in  $S$ .
- The Cartesian product of two sets  $A$  and  $B$ , is the set of all possible pairs, where one element is from  $A$  and the other from  $B$ .
- *Understand Cantor's diagonalization proof.*