Crib 1

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

1 Propositional Logic

- Apply DeMorgan's to move negations past $\wedge, \vee$. Negate both clauses, and swap and for or and vice versa.

$$\neg(P \wedge Q) = \neg P \vee \neg Q$$
$$\neg(P \vee Q) = \neg P \wedge \neg P$$

- To move negations past quantifiers, switch quantifiers.

$$\neg \forall x \in \mathbb{Z} P(x) = \exists x \in \mathbb{Z} \neg P(x)$$
$$\neg \exists x \in \mathbb{Z} P(x) = \forall x \in \mathbb{Z} \neg P(x)$$

- $\forall x \exists y P(x, y) \neq \exists y \forall x P(x, y)$. (For every student in this room, there exists a pair of pants, but it is not true that there exists a pair of pants for all students in this room.) However, the latter *does* imply the former.
- Remember the conjunctive normal form (CNF) of implications: $P \implies Q = \neg P \vee Q$.
- Do not move the quantifier past the operator if the involved variable is on both sides of the operator. For example, we know that $(\exists y \in \mathbb{Z}, y < 0) \wedge (\exists y \in \mathbb{Z}, y \geq 0)$ is true, but if we move the quantifier left of the operator, we get a false statement $\exists y \in \mathbb{Z}(y < 0 \wedge y \geq 0)$. This is false because instead of choosing two separate ys, we are now picking one y for both clauses.
- Feel free to move the quantifier past the operator if the involved variable is *not* on both sides of the operator. i.e., For $\exists x, y \in Z, P(x) \wedge Q(y)$, you can move $\exists y$ to get $\exists x \in Z, P(x) \wedge (\exists y \in Z, Q(y))$.

2 Proofs

- In a *direct proof* for a statement Q with a truth P, we show that $P \implies Q$, where P is some known and Q is the claim we want to prove.
- In a *proof by contraposition* for a statement $P \implies Q$, prove $\neg Q \implies \neg P$.
- In a *proof by contradiction* for a statement P, assume for sake of contradiction that $\neg P$ is true. Show that this leads to $R \wedge \neg R$ (a contradiction, by the Law of Excluded of Middle). This means P is true.
- Here is a common mistake: In a proof by contradiction, you must show $R \wedge \neg R$. In other words, $\neg P$ implies something that defies mathematical laws. This consequently, means that $\neg P$ cannot be true, so P is true. (Proof by contradiction is *different* from a counterexample. Do not "disprove" $\neg P$ using a counterexample; you can't "disprove" a propositional statement using just a counterexample anyhow).