

Quiz 21 Solutions

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam.

1 Watermelon Eating

Watermelon is the world's best fruit. Since it's the world's best fruit, I am determined to eat more and more of it each day. Assume I can buy at most 5 watermelon slices in a day. If I randomly choose a number of watermelon slices to eat each day, how many slices do I eat, with 95% confidence?

Solution: We can immediately use Chebyshev's. First, let us express the question mathematically, where k is the number of slices above average. Let X be the number of watermelon slices I eat.

$$X - \mu \geq k$$

For now, we will take the magnitude. Later, we will amend this issue.

$$|X - \mu| \geq k$$

By Chebyshev's, we know the following.

$$P[|X - \mu| \geq k] \leq \frac{\text{var}(X)}{k^2} = 0.95$$

Since, X is chosen at random, it is uniformly distributed. Additionally, we know that there exists a lower bound of 0 and an upper bound of 5. The formula for variance of a uniform distribution is $\frac{(b-a+1)^2-1}{12}$, making the variance $\frac{(5-1+1)^2-1}{12} = \frac{5^2-1}{12} = \frac{24}{12} = 2$.

Thus, we have the following:

$$\frac{2}{k^2} = 0.95$$

However, keep in mind that we are only interested in the number of slices *above average*, that we can claim to eat. Chebyshev's considers probability of being more than $\mu + \epsilon$ *or* being less than $\mu - \epsilon$. Since we're only interested in one of two cases, we divide the probability by 2.

$$\frac{2}{k^2} = 0.475$$

$$\frac{2}{0.475} = k^2$$

$$k \sim 2$$