Quiz 20 Solutions

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Thursday, April 7, 2016

This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. In this quiz, we will walk through various sorts of independence.

1 Determining Independence

1. Let X and Y be the number of pips rolled for two different dice. What is $E[XY]$? $P[X = 1, Y = 3]$?

   Solution: $\frac{49}{4}$, $\frac{1}{36}$ Since X and Y are independent, we know that

   $$E[XY] = E[X]E[Y]$$

   $$P[XY] = P[X]P[Y]$$

   Note: The converse is *not* true for expectation. $E[XY] = E[X]E[Y]$ does *not* imply independence. However, independence does imply that relationship.

2. Compute $var(X)$, where X is the number of heads after n flips of a biased coin with heads-probability p.

   Solution:

   Recognizing a Distribution

   We know that we need an indicator, X_i which is flipping a head on the ith trial. Since all X_i are independent, we can define $X \sim Bin(n, p)$. Thus, the variance is $np(1 - p)$. This makes your life very easy.

   Full Derivation

   To demonstrate this, the following is a full derivation of this variance:

   $P[X_i] = p$ and by virtue of it being an indicator variable, $E[X_i] = p$ as well.

Since all coin flips are independent, we can apply linearity of variance.

$$var(X) = var(\sum_i X_i) = \sum_i var(X_i) = nvar(X_i)$$

Now, we simply need to compute $var(X_i)$. Note that $E[X_i^2] = E[X_i]$ in this case, since we are considering an indicator variable X_i.

$$\begin{aligned} var(X_i) &= E[X_i^2] - E[X_i]^2 \\ &= p - p^2 \\ &= p(1-p) \end{aligned}$$

Thus, $var(X) = nvar(X_i) = np(1-p)$.