Quiz 19 Solutions

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. In this quiz, we will walk through identifying distributions.

1 Identifying Distributions

For each of the following questions, identify the distribution and specify the parameters. For example, the number of heads in n coin flips is $X \sim Bin(n, \frac{1}{2})$.

1. Whether or not you roll a number greater than 4 (given a normal 6-sided die).

   Solution: $X_i \sim Bernoulli(\frac{1}{3})$

2. The number of times you roll a number greater than 4 in n flips.

   Solution: $X \sim Bin(n, \frac{1}{3})$

3. The number of times 3 rolls roll exactly $\{3, 4, 5\}$, in n flips.

   Solution: $X \sim Bin(n-2, \frac{1}{6^3})$. Define an indicator variable X_i which is whether or not we have ended a sequence of $\{3, 4, 5\}$ at the i-th roll. Note that we can't start a sequence of 3, in the last roll or the second-to-last roll. So, our number of trials is actually $n-2$.

4. The number of times you expect to roll, until you achieve a number greater than 4.

   Solution: $X \sim Geom(\frac{1}{3})$

5. The average amount of ice cream eaten in the summer - in pounds - given the average for June is 1, July is 2, and August is 3.

   More rigorously: Distribution of A, given $A = X + Y + Z$, and $X \sim Poiss(1), Y \sim Poiss(2), Z \sim Poiss(3)$.

Solution: $A \sim Poiss(6)$. Poisson distributions summed, give another Poisson distribution where the averages are summed.

6. The number of times 3 rolls in sequence are all numbers greater than 3, in n flips.

   Solution: $X \sim Bin(n-2, \frac{1}{2^3})$. Same reasoning as 4.

7. $min(G_1, G_2, ...G_n)$ where G_1 through G_n are all geometric distributions with parameters p_1 to p_n.

   Solution: $X \sim Geom(f(\{p_1, p_2...p_n\}))$

8. Taking $X \sim Bin(n, p)$ and defining a new random variable Y, such that $E[Y] = np$ and $\sigma^2 = np(1-p)$

   Solution: $X \sim Normal(np, np(1-p))$