Quiz 13 Solutions

written by Alvin Wan . alvinwan.com/cs70

Tuesday, March 8, 2016

This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. In this quiz, we will walk through counting with problems of increasing difficulty.

1 Counting (with Ice Cream)

1. How many ways are there to pick 3 scoops of 5 possible ice creams each?

   Solution: 5^3. This is *sampling with replacement*, as we're "putting flavors back into the bag, " so that after picking an ice cream flavor, a scoop picked later can also have the same flavor.

2. Let us now order all 5 ice cream flavors. How many ways are there to pick 3 scoops such that the order is respected? (No repeating scoops.)

   Solution: $\binom{5}{3}$. Intuitively, we can view this as the following: Pick 3 flavors from the set of 5, and there is only one way to order all 3. Thus, the set of all possible ice cream configurations is strictly the set of all combinations.

   How Ordering Affects the Problem Any restriction on the samples we choose that enforces a similar "ordering" or property will reduce the set of all permutations to the set of all combinations.

3. How many ways are there to sprinkle 10 oreo pieces on our 3 scoops of ice cream?

   Solution: $\binom{12}{2}$ This reduces to a stars and bars problem, where we have 3 possible "buckets" (2 bars)and 10 stars.

   Stars and Bars How?: Note that to specify 3 buckets, we need only 2 bars to separate the 10 stars. Thus, we can choose the 2 bars or the 10 balls from 12 slots.

We will explore more of these types of problems, next discussion.