

# Quiz 6

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**This quiz does not count towards your grade.** It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. "Intuition Practice" might be tricky; watch out for subtleties. "Proofs" will be challenging to start; develop an arsenal of *approaches* to starting a problem.

## 1 Proofs

1. **Strongly-connected Components** A strongly-connected component of a graph is a set of vertices  $S$  where each pair of vertices  $(u, v) \in S$  is connected by a path. Prove that a connected graph with directed edges cannot contain SCCs if there are fewer than  $|V|$  edges.

### 2. Hypertriangles

Let us define a hypertriangle, where the dimension 1-dimensional hypertriangle is the triangle. Hypertriangles are defined recursively, as hypercubes are, except each  $n$ -dimensional hypertriangle is defined using 3  $(n-1)$ -dimensional hypertriangles.

- (a) Prove that the  $n$ -dimensional hypertriangle consists of  $3^n$  vertices.
- (b) Prove that the degree of all vertices in the hypertriangle are the same.
- (c) Prove that the vertices' degree is always even.
- (d) Find the number of edges in an  $n$ -dimensional hypertriangle and justify your answer.