Quiz 4 Solutions

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. "Intuition Practice" might be tricky; watch out for subtleties. "Proofs" will be challenging to start; develop an arsenal of *approaches* to starting a problem.

1. Prove that an undirected, connected graph with $|V|$ edges must contain a cycle.

   How do I start?

   We could approach this using either induction or contradiction. Contradiction is far shorter and easier. Although if you do induction, make sure to not commit "Build-up Error".

   Proof by Contradiction

   Assume for contradiction an acyclic, connected graph of $|V|$ edges exists. There are two possibilities for an acyclic graph. It is either (1) minimally connected or (2) disconnected. The graph cannot be disconnected, so we only consider the first option. We know that a minimally connected graph is a tree. All trees have $|V| - 1$ edges. Contradiction.

2. Let a vertex with 0 in-degree and non-zero out-degree be a source. Let a vertex with 0 out-degree and non-zero in-degree be a source. Prove that any graph with an Eulerian tour cannot contain a source or a sink.

   How do I start?

   Notice the negation in the statement we are trying to prove. There isn't much we can say about *not* containing a source or a sink. We have information about the negation, however - a graph that *contains* a source or a sink. So, we know that we should contradiction or contraposition.

   Proof by Contraposition A graph with a source or a sink will result in a vertex that can only be entered or only be exited. This means that once entered or exited cannot return to the node or to the rest of the graph. This means a tour cannot be completed. QED.