

# Quiz 3 Solutions

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**This quiz does not count towards your grade.** It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. "Intuition Practice" might be tricky; watch out for subtleties. "Proofs" will be challenging to start; develop an arsenal of *approaches* to starting a problem.

## 1 Intuition Practice

The following questions apply to a modified Stable Marriage Algorithm ("MSMA"). In MSMA,  $n_c$  candidates are applying to  $n_s$  schools, where each college maintains a waitlist for and accepts  $x_i$  candidates. We know  $\sum_{i=0}^{n_s} x_i = n_c$  and that all other facets of the Stable Marriage Algorithm are preserved. Write down only "True" or "False". Do not justify your answers.

1. MSMA is college-optimal and student-pessimal.  
**False.** The side proposing maintains optimality. In this case, the students are applying, or "proposing", so it is student-optimal and college-pessimal.
2. MSMA always yields a stable pairing.  
**True.** We can apply SMA's proof for stability to MSMA.
3. Let us define  $w_k$  to be the number of students college  $w$  has on the waitlist on day  $k$ . On arbitrary days  $i, j$  such that  $i < j$ ,  $w_i > w_j$ .  
**False.** By the improvement lemma, the waitlist can only grow in number of students.
4. MSMA is guaranteed to terminate in  $\leq n_s$  days.  
**False.** One student  $s_1$  could spend  $n_s - 2$  days making futile proposals.  $s_1$  could then "kick"  $s_2$  from the second-to-last college, who may make another  $n_s - 1$  proposals to settle.
5. It is possible for all colleges to end up with the last  $x_i$  students on its preference list.  
**True.** Let the final  $x_i$  students on all college  $c_i$ 's lists could be exclusive, meaning no two colleges have the same student in the last  $x_i$  of the preference list. If all students in those final  $x_i$  list that college  $c_i$  as their first choices, then MSMA terminates in one day, where each college has the bottom  $x_i$  of its preference list.