Quiz 1

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam. "Intuition Practice" might be tricky; watch out for subtleties. "Proofs" will be challenging to start; develop an arsenal of *approaches* to starting a problem.

1 Intuition Practice

For the following, we assume $\forall x \in \mathbb{Z}, A(x) \wedge B(x) \implies \exists y \in \mathbb{Z}, C(x,y)$. Your options are True or False.

1. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, C(x,y)$
2. $\forall x \in \mathbb{Z}, A(x) \wedge B(x) \iff \forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, C(x,y)$
3. $\forall x \in \mathbb{Z}, \exists y \in \mathbb{Z}, \neg A(x) \vee \neg B(x) \vee C(x,y)$

2 Proofs

1. Prove or disprove that $\forall n \in \mathbb{Z}, \exists k \in \mathbb{Z}, n^3 + 3n^2 + 2n = 3k$.

2. Consider 2×2 "L"-shaped tiles. Prove that if a board has dimension $w \times h$, such that $w, h \in \mathbb{Z}, (\forall k \in \mathbb{Z}, h \neq 3k) \wedge (\exists k \in \mathbb{Z}, \forall l \in \mathbb{Z}, w = 2k \neq 3l)$, the board has no perfect tiling. Define a "perfect tiling" to be a configuration of L-shaped tiles, where no block on the board is left uncovered and each slot contains at most one tile.