Quiz 19 : Markov Chain Concepts Solutions

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This quiz does not count towards your grade. It exists to simply gauge your understanding. Treat this as though it were a portion of your midterm or final exam.

1 Definitions

Let X_i denote state i in a Markov chain.

1. (**True** or **False**) Markov chains can have more than one invariant distribution.

   Solution: True. Consider a two-node Markov Chain with states X_1, X_2, where the transition matrix is $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$. Note that any initial distribution is its own invariant distribution.

2. (**True** or **False**) It is necessarily true that X_{i+1} and X_{i-1} are uncorrelated.

   Solution: False. X_{i+1} only depends on X_i and is *conditionally* independent of X_{i-1} but we do not know that X_{i+1} and X_{i-1} are independent.

3. (**True** or **False**) It is possible that $E[X_{i+1}X_i] = E[X_{i+1}]E[X_i]$. *Hint: Does X_{i+1} depend on X_i? Does it matter?*

   Solution: True. Though X_{i+1} depends on X_i, we note that independent X, Y implies $E[XY] = E[X]E[Y]$. The converse does not hold, so it is possible for dependent X, Y to satisfy $E[XY] = E[X]E[Y]$.

2 Corollaries

For the following questions, provide a brief justification. Consider the following transition matrix.

$$P = \begin{bmatrix} 1 & a & 0 \\ a & 1-a & 0 \\ 0 & 0 & a \end{bmatrix}$$

1. (**True** or **False**) For P above, there exists an irreducible, periodic Markov Chain.

   Solution: False. In order for the Markov Chain to be valid, all columns must sum to 1. Per column 3, a must be 1. Per column 1, a must be 0. Contradiction.

2. (**True** or **False**) There exists an irreducible, aperiodic Markov chains without a unique invariant distribution.

   Solution: False. Irreducibility implies the existence and uniqueness of a Markov Chain.