Quantifiers

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1 Misconceptions

First, quantifiers do not obey some universal law of distributivity. They also do not obey some universal law of associativity. This sheet summarizes some of these rules.

2 "Associativity"

- **Universal quantifiers can be swapped with each other.** $\forall x \forall y, P(x, y) \equiv \forall y \forall x, P(x, y)$
- **Universal quantifiers cannot be swapped with existential quantifiers.** $\forall x \exists y, P(x, y) \neq \exists y \forall x, P(x, y)$
  1. $\forall x \exists y, P(x, y) \not\Longrightarrow \exists y \forall x, P(x, y)$
     Consider the following counterexample, showing the left-hand side (LHS) is not the same as the right-hand side (RHS) "For all students, there exists a pair of pants, but it is (hopefully) not true that there exists a pair of pants for all students."
  2. $\exists y \forall x, P(x, y) \implies \forall x \exists y, P(x, y)$
- **Existential quantifiers can be swapped with each other**. $\exists x \exists y, P(x, y) \equiv \exists y \exists x, P(x, y)$

3 "Distributivity"

- $\forall x \forall y (P(x, y) \wedge Q(x, y)) \equiv \forall x (\forall y, P(x, y)) \wedge (\forall y, Q(x, y))$
- $\exists x \exists y (P(x, y) \vee Q(x, y)) \vee \equiv \exists x (\exists y P(x, y)) \vee (\exists y Q(x, y))$
- $\exists x \forall y (P(x, y) \vee Q(x, y)) \vee \not\equiv \exists x (\forall y P(x, y)) \vee (\forall y Q(x, y))$