Midterm 1 : Strategies and Tips

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This sheet covers direct proofs, proof by induction, proof by contradiction, and proof by contraposition for stable marriage, graph theory, and modular arithmetic.

1 Exam-Taking Tips

- Feeling *extremely* nervous? Take 7 seconds to inhale, hold your breath for 7 seconds, and take 7 seconds to exhale.

- Feel a *little* nervous? It's normal, and it's good to be on your toes a little bit.

- Don't panic. If a problem doesn't initially make sense, read through slowly and digest it. You can do it.

2 General Approach to a Proof

1. Name the items in your proof. i.e., "Let man M's optimal woman be W" or "Consider an integer $z \in Z$"

2. Convince yourself that the proof is true, or find a counterexample if your intuition says otherwise. Draw diagrams and graphs if need be.

3. Draw upon previously-proved theorems and lemmas. If you don't know where you're going, explore every avenue that you can.

4. Make it bullet proof. Check that you're not committing a logical fallacy: begging the question, circular tautology etc. Make sure to check for all cases.

5. Explain the proof to your rubber duck. (Or imaginary rubber duck.) The idea is that you may catch a blatant error, when you stumble in your explanation.

3 Propositional Logic

Before you begin, ask yourself: Is this even a proposition?

- Remember that quantifiers cannot be switched arbitrarily. See Crib 01 for rules.

- Propositional logic can be your best friend; given a complex question with many moving parts, it may be beneficial to write the propositional statement. (It's easier to see the contrapositive).

- Quantifiers may sometimes belong to the proposition. e.g., If there exists a pup, there exists a mother dog. Let $A(x)$ denote the age of a given dog x and $M(x, y)$ is true if y is the mother of x. $(\exists p, A(p) < 1) \implies (\exists P, A(P) \geq 1 \land M(x, y))$. Note that moving the quantifiers out would have a different meaning: There exists two dogs. If one is a pup, the other is the mother.

4 Induction

Induction relies on the fact that there exists an orderable set. You can always induct on the *number* of some item (vertex, edge, matchings, integers etc.)

- Remember to avoid committing build-up error. See Crib 02.

- Strengthen your inductive hypothesis if it "would be nice" to have some form.

- Consider strong induction if your proof for $P(n + 1)$ relies on more than $P(n)$.

5 Stable Marriage

Most stable marriage proofs use contradiction and the well-ordering principle. See Crib 03 for more details.

6 Graph Theory

Most graph theory proofs use induction or contradiction. It is highly recommended that you attempt induction on vertices first, before trying the others. See Crib 04 for details.

- Understanding the definitions and remembering the implications of each fact (e edges means $2e$ degrees, n-dimensional hypercube means 2^n vertices etc.) are critical.

- Especially for graph theory, consider the lemmas and theorems you have available to you, not to mention the equations and inequalities that have already been proven in the notes.