Crib 21 : Continuous Probability

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

1 Tactics

- In continuous probability, all summations are integrals. For example, take the following discrete concepts:
  - $E[X] = \sum xP(X = x)$
  - PMF: $P(X = k)$, CDF: $\sum_k P(X = k)$

  Take the following continuous analogs:
  - $E[X] = \int xP(x = x)$
  - PDF: $P(x = k)$, CDF $= \int P(X = k)dk$

  Note that the PDF is a continuous-valued function, whereas the PMF is a function defined only at discrete points.

- Drawing is an important tactic. Take your two random variables, and consider all of their possible combinations of values. Then, draw the regions over which you're interested in.
  - For uniformly-distributed random variables, the ratio of the area of your region to the entire region, is the probability of that event. This is true because a uniform distribution has a joint PDF inversely proportional to the entire area.
  - For non-uniformly-distributed random variables, integrate the joint PDF $f_{X,Y}(x, y)$ over your region of interest.

- We have the following analogs for discrete v. continuous distributions.

- The **binomial distribution** handles n independent trials with probability p of success. It answers *what is the probability of k successes in n trials?*. Likewise, if $np \leq 1$, we see that the **Poisson distribution** is a fair approximation of binomial. The Poisson distribution handles an average number of successes λ per unit time. Poisson answers *what is the probability of k successes per unit time?*.
- The **geometric distribution** handles independent trials with probability p of success. It answers *what is the amount of time until the first success?*. Again, our limiting distribution has an analog; take the limit of increasingly shorter units of time to get the continuous **exponential distribution**. Exponential distribution handles again the average number of successes λ per unit time. However, it answers *how many units of time until the first success?*.

2 Notes

- It is important to note that the uniform distribution is defined for both discrete and continuous-valued random variables.
- There are various ways to combine random variables:
  - The sum of Poisson random variables $P_i \sim \textsc{Pois}(\lambda_i)$ is another Poisson-distributed random variable with parameter $\sum_i \lambda_i$.
  - The minimum of exponential random variables $E_i \sim \textsc{Expo}(\lambda_i)$ is another exponentially-distributed random variable with parameter $\sum_i \lambda_i$.
  - The sum of Gaussian random variables $N_i \sim N(\mu_i, \sigma_i^2)$ is another Gaussian random variable with parameters $N(\sum_i \mu_i, \sum_i \sigma_i^2)$
  - Remember to always specify the valid values for your random variable when writing a PDF.