Crib 20 : Markov Chain Problems

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

1 Tactics

- To solve a **hitting time** problem, write out your balance equations and solve the linear system of equations. Remember that we consider the set of possible *destinations*. In other words,

  $$\beta(i) = p\beta(i-1) + (1-p)\beta(i-2) + 1$$

  For the above equation, this means that from state i we have probability p of reaching state $i-1$ next and probability $1-p$ of reaching state $i-2$. We assume that this transition takes "time" 1. Note that this transition could take a variable amount of time. In which case, add the amount of time the transition takes, instead of 1.

- To solve for the **invariant distribution**, do the following:

  1. Check that the Markov Chain is irreducible. Only then can we guarantee that there exists a unique invariant distribution. Note that invariant distributions may exist for other Markov Chains but *are not guaranteed.*
  2. Write out all balance equations. As it turns out, this system of equations is dependent.
  3. So, replace one equation with the requirement that all $\pi(i)$ sum to 1. Explicitly, remove an arbitrary balance equation, and add the equation $\pi(1) + \cdots + \pi(n) = \sum_{i=1}^{n} \pi(i) = 1$.
  4. Write the balance equations as a matrix P. Solve the augmented matrix $[P|\pi]$.

- To solve **probability of A before B**, consider the probability of reaching any state in A from states neither in A nor in B. Then, consider the probability of reaching B from states in A. We again consider the set of possible *destinations*. Note there is no extra term.

$$\alpha(i) = p\alpha(i-1) + (1-p)\alpha(i-2)$$

  The above means that from state i, we have probability p of reaching state $i-1$ and probability $1-p$ of reaching $i-2$.

- Recall the definition of a Markov Chain; it models only a series of states that depend on the *current* time step. States cannot have dependence on multiple prior time steps.

- All values must sum to 1.

$$\sum_i \pi(i) = 1$$

- All transition probabilities for a single destination must sum to 1.

$$\sum_j \Pr(i,j) = 1$$