Crib 19 : Markov Chain Concepts

written by Alvin Wan . alvinwan.com/cs70 . Wednesday, November 9, 2016

The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

- A Markov chain is a set of states X_i, where each X_{i+1} only depends on X_i. In this class, we only consider Markov Chains with finitely many states.
- P is the **transition matrix**. $P(i, j)$ gives us the probability of a transition from X_i to X_j.
- The current state of a Markov Chain (i.e., values at all the nodes) at time t is represented using π_t. So, $\pi_{t+1} = \pi_t P$.
- A Markov Chain is **irreducible** iff there exists some path between every pair of states. (i.e., for each state, all other states are "reachable"). Note this means the Markov Chain must be a single connected component.
- A state X_i is **aperiodic** if the length of all paths starting at X_i and ending at X_i has GCD 1. More formally,

  $$d(i) := \text{G.C.D.}\{n > 0 | P^n(i, i) = \Pr(X_n - i | X_0 = i) > 0\}, i \in X$$

  X_i is aperiodic if $d(i) = 1$.
- A Markov Chain is aperiodic if all of its states are aperiodic.
- An **invariant distribution** π with transition matrix P is a distribution such that $\pi = \pi P$.
- An irreducible Markov Chain always has a unique invariant distribution.
- **Balance equations** specify transitions for a Markov Chain. Let $\pi(j)$ denote the value of state j. We express $\pi(j)$ in terms of all possible paths from i to j. So $\pi(j) = \sum_i P(i, j)\pi(i)$.