Crib 18 : Conditional Expectation

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

- Formally, we have that $E[X|Y]$ is the following, for discrete variables.

$$E[X|Y] = \sum_x x \Pr(X = x|Y)$$

- The **Law of Iterated Expectations** states that, for any random variables X, Y,

$$E[E[X|Y]] = E[X]$$

- For random variables X, Y, the **Law of Total Expectation** states that

$$E[X] = \sum_y E[X|Y = y] \Pr(Y = y)$$

- We can consider conditional $E[X|Y]$ to be a function of Y. Note that $E[X|Y]$ *is a random variable*, unlike $E[X]$ or $E[Y]$.

- We can thus model the evolution of a system over time using $E[X_t|X_{t-1}]$. In other words, we can express the state at some time step t, X_t with a function g in terms of the state at the previous time step, $g(X_{t-1})$.

- Consider $X(t+1) = \alpha X(t)$ for some constant α. In terms of $X(0)$, we have that $X(t)$ is

$$X(t) = \alpha^t X(0)$$

- Consider $X(t+1) = \alpha X(t) + \beta$ for constants α, β. In terms of $X(0)$, $X(t)$ is

$$X(t) = \alpha^t X(0) + \beta(\sum_{i=0}^{t-1} \alpha^i)$$

(Note that the summation begins from $a^0 = 1$)