Crib 12

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

1 Probability

- Ω is the sample space. $\Pr(\Omega) = 1$
- An **outcome** is a point in the sample space.
- An **event** is a set of outcomes.
- **Inclusion-exclusion** tells us that $\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$.

  On a side note: We also denote $\Pr(A \cap B)$ as $\Pr(A, B)$.

  Hint: Use Venn Diagrams to convince yourself of both this statement and the generalized statement for inclusion-exclusion.
- The **Law of Total Probability** implies that $\Pr(A) = \Pr(A, B) + \Pr(A, \bar{B}) = \Pr(A|B)\Pr(B) + \Pr(A|\bar{B})\Pr(\bar{B})$.

  Full statement: $\Pr(A) = \sum_B \Pr(A, B) = \sum_B \Pr(A|B)\Pr(B)$
- The **Chain Rule** implies that $\Pr(A, B) = \Pr(A|B)\Pr(B)$.

  Full statement: $\Pr(X_1, X_2 ... X_n) = \Pr(X_1|X_2 ... X_n) ... \Pr(X_{n-1}|X_n)\Pr(X_n)$
- Common trick: For probability, we can count the number of combinations that satisfy a condition and divide by the number of total combinations.
- Another common trick: To count probability of "at least one success", we can consider 1− probability of "no successes". For example, given n independent trials, if we have probability p of *success*, then we have $(1 - p)$ probability of failure. The probability of failing every time is $(1 - p)^n$. The probability of at least one success is thus $1 - (1 - p)^n$.