Crib 4

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

1 Definitions

- The **degree** of a vertex is the number of edges incident to it. For a directed graph, out degree is the number of outgoing edges incident to a vertex and in degree is the number of incoming edges incident to a vertex.
- A **path** is any sequence of edges, where each edge is connected to at most two other edges such that no vertex is repeated.
- A **cycle** is any path that starts and ends at the same vertex.
- A **walk** is a more general version of a path where vertices *can* repeat.
- A **tour** is any walk that starts and ends at the same vertex.
- A **complete** graph is one where all vertices have maximum degree.
- An **Eulerian Tour** is a tour that visits all edges in a graph exactly once. *This means an Eulerian Tour cannot repeat edges but can repeat vertices.*
- A **Hamiltonian Cycle** is a cycle that visits all vertices exactly once. *This means a Hamiltonian Cycle cannot repeat vertices.*
- A **Tree** is defined by the following four properties. *In this course, we deal with undirected trees.* These are all equivalent conditions (i.e., If one holds, they all hold.):
  - A connected graph with $|V|$ vertices and $|V| - 1$ edges.
  - A connected graph with no cycles.
  - A connected graph, where removing an edge disconnects the graph.
  - An acyclic graph, where adding an edge anywhere creates a cycle.
- A **hypercube** in the nth dimension is constructed by joining two hypercubes of dimension $n - 1$.

Here are some interesting implications of these definitions. Take a moment to think about the following, as building these intuitions yourself is critical to developing further intuition, on your exams.

- A path is a walk, but a walk is not a path.
- A cycle is a tour, but a tour is not a cycle.
- An Eulerian Walk, like an Eulerian Tour, covers all edges. However, an Eulerian walk does not have to start and end at the same point, as an Eulerian Tour does.
- A Hamiltonian Path, like a Hamiltonian Cycle, covers all vertices. However, a Hamiltonian path does not have to start and end at the same point, as a Hamiltonian Cycle does.

2 Planarity

- A K_n graph is a complete graph with n vertices.
- A $K_{a,b}$ graph is a bipartite graph where one side has a vertices and the other side has b vertices.
- A graph is planar if and only if it does not have the K_5 and $K_{3,3}$ minors.
- The number of faces is the number of cycles.
- Euler's formula states that $v + f = e + 2$.

3 Hypercubes

- A hypercube in dimension n has $n2^{n-1}$ edges and 2^n vertices.
- To construct the $n+1$th dimension hypercube, combine two n-dimension hypercubes by connecting corresponding vertices.

4 Tips

- **Watch out for build-up error.** Proper induction must be done the following way: Start with the $k+1$th step, and reduce to the kth step, so that you can apply the inductive hypothesis. Then, build back up to the $k+1$th step.
- Remember that an Eulerian Tour can only be constructed if and only if all vertices in a graph have even degree, and the graph is connected.