

Crib 2

written by Alvin Wan . alvinwan.com/cs70

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The crib sheet contains cheat-sheet worthy information but is not a substitute for lectures or for reading the notes. It also contains pointers and common mistakes.

1 Induction

- **Induction is comprised of three steps:** (1) base case, (2) inductive hypothesis, and (3) induction step. Explicitly identify each on your homework and exams.
- There are several ways to prove your inductive step:
 - Take your $k + 1$ case, **break it down to the k case, apply the inductive hypothesis, and then re-extend it** to the $k + 1$ case. This is necessary later on, in graph theory, and is the proper way to conduct an induction proof.
 - If your proof is entirely algebraic: To prove the $k + 1$ case, it is generally better to **plug in $k + 1$ on the left-hand side and show how to get the right-hand side** *without modifying both sides at the same time*. This makes your proof clearer. Feel free to modify both sides in scratch work, but in your final answer, it's better to move only in one direction. The reason why this suffices for algebra-only proofs is that algebra is symmetric.
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- **Make sure to cover all possible base cases.** Consider the following false statement $\forall n \in \mathbb{N}, 2^n \geq 3^n$. We could consider the base case to be $n = 0$ since the smallest element of \mathbb{N} is 0, and proceed with our base case: *Plug in $n = 0$ and get $2^0 \geq 3^0, 1 = 1$, which is true. Base case proved.* Of course, this is false though. Before wasting time on the inductive step, we should have tried base case $n = 1$. We can tell another base case is needed: $n = 0$ only proves the case where $LHS = RHS$ but not where $LHS \geq RHS$.
- **Strong induction is weak induction and vice versa.** Consider the following transformation from a strong induction proof (S) to one using weak induction (W). Say in S, our inductive hypothesis is "For all $1 < k' < k, P(k')$ ". We can simply create a new proposition " $Q(k) = \forall k', 1 < k' < k, P(k')$ " and apply weak induction in W.