Quiz 4

04 Gaussian Discriminant Analysis, Decompositions

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For the multiple choice questions, select *all* that apply.

1 Gaussian Discriminant Analysis

The following algorithms will yield a decision boundary even with data that is not linearly separable.

(a) Linear Discriminant Analysis

(b) Quadratic Discriminant Analysis

(c) Perceptrons

(d) Soft-Margin Support Vector Machine

Solution: All but c, which will not terminate if the data is not linearly separable. Although LDA produces a linear decision boundary, it simply computes a value.

The following always produces a linear decision boundary, regardless of the data provided to it.

(a) Linear Discriminant Analysis

(b) Quadratic Discriminant Analysis

(c) Perceptrons

(d) Hard-Margin Support Vector Machine

Solution: Only the a) is guaranteed to produce a linear decision boundary. b) produces quadric surfaces and d) potentially creates extremely complex decision boundaries. c) might not converge, thus not producing a decision boundary at all, much less a linear one.

2 Decompositions

Prove that if v_i with eigenvalue λ_i is an eigenvector for a symmetric A, it is also an eigenvector for the outer product of $A - \lambda I$.

Solution:

$$\begin{aligned}(A-\lambda I)(A-\lambda I)^T v &= (AA^T - 2\lambda A + \lambda^2)v \\ &= AAv - 2\lambda Av + \lambda^2 v \\ &= \lambda_i^2 v - 2\lambda_i \lambda v + \lambda^2 v \\ &= (\lambda_i - \lambda)^2 v\end{aligned}$$

Consider a real, symmetric A, which admits an eigendecomposition. Prove that $\|A\|_F = \|\lambda\|_2$, where $\lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]^T$ for eigenvalues λ_i of A.

Solution: We square both sides. Consider the eigendecomposition of $A = PDP^T$.

$$\begin{aligned}\|A\|_F^2 &= \mathrm{Tr}(A^T A) \\ &= \mathrm{Tr}(PD^2P^T) \\ &= \mathrm{Tr}(P^T PD^2) \\ &= \mathrm{Tr}(D^2) \\ &= \sum_i D_{ii}^2 \\ &= \sum_i \lambda_i^2 \\ &= \|\lambda\|_2^2\end{aligned}$$