Quiz 2

02 Perceptrons

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Treat this as an exam situation. You will be given 5 minutes to complete this quiz.

1 L2 Norm

Prove that the L2 norm is unitary invariant. In other words, the L2 norm of a vector does not changed even after multiplying by some orthogonal matrix U.

Solution: Consider an orthonormal matrix $U \in \mathbb{R}^{d\times d}$. Recall that $U^TU = I$ since the column vectors of U are by definition linearly independent and normalized. This means all $i \neq j$, $u_i^T u_j = 0$ and all $i = j$, $u_i^T u_j = 1$. Our goal is to show that for all vectors $v \in \mathbb{R}^d$, $\|Uv\|_2^2 = \|v\|_2^2$.

$$\|Uv\|_2^2 = (Uv)^T(Uv) = v^TU^TUv = v^Tv = \|v\|_2^2$$

2 Distance to Hyperplane

For a point $x_i \in \mathbb{R}^d$, prove that the distance to a hyperplane $H = \{x : w^Tx + \alpha\}$ is

$$\frac{1}{\|w\|_2}(w^Tx_i + \alpha)$$

Solution: This is proved in Note 2, restated as Theorem 1 in Section 1. Click below to access it:

aaalv.in/cs189/sp17/notes/n2.pdf